OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **2641**

Probability & Statistics 1

Tuesday **28 MAY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Two directors of a company each interviewed seven candidates for a job. Director 1 gave each candidate a score from 0 to 100 and a high score meant that the director liked the candidate. Director 2 ranked the candidates and gave the candidate she liked most a rank of 7 and the candidate she liked least a rank of 1. The results are given in the table below.

Candidate	A	B	C	D	E	F	G
Director 1	74	83	64	58	27	91	28
Director 2	6	5	2	4	3	7	1

(i) Calculate Spearman's rank correlation coefficient between the assessments given by Director 1 and Director 2. [5]

(ii) What can be said about the amount of agreement between the two directors in their assessment of the 7 candidates? [1]

2 A discrete random variable X has the probability distribution given by the table below. The expectation of this distribution is denoted by μ and the variance is denoted by σ^2.

x	2	3	4	5	6
$\mathrm{P}(X = x)$	0.2	0.1	0.1	0.4	p

(i) Show that the value of p is 0.2. [2]

(ii) Find μ. [2]

(iii) Find σ^2. [3]

3 Three married couples, Mr and Mrs Aziz, Mr and Mrs Baker and Mr and Mrs Campbell, are arranged in a line for a photograph.

(i) How many different arrangements of the six people are possible? [1]

(ii) In how many of these arrangements is Mr Aziz standing next to his wife? [4]

(iii) Given that every possible arrangement is equally likely, calculate the probability that Mr Aziz is standing next to his wife. [2]

4 A jar contains 4 red discs, 6 white discs and 5 green discs. Three discs are removed at random from the jar, one after the other. Once a disc has been removed it is not replaced in the jar.

(a) Find the probability that

(i) the first disc is red, [1]

(ii) the second disc is white if the first disc is green. [1]

(b) Calculate the probability that

(i) exactly two of the three discs are red, [3]

(ii) the three discs are the same colour. [3]

5 On a fairground stall the prize is a large cuddly toy worth £20. In order to win this prize Vicky needs to throw a hoop over a pole. For each throw, her probability of success is $\frac{2}{35}$, independently of all other throws. She pays the owner of the stall 50p for each throw and keeps throwing the hoop until she wins the prize.

(i) Calculate the probability that Vicky takes fewer than 3 throws to win the prize. [3]

(ii) Find the expected number of throws that Vicky needs to win the prize. [2]

(iii) Calculate the probability that Vicky pays more to win the prize than it is worth. [3]

6 Two chess players, A and B, play in a tournament. Each player plays against 11 different opponents. Each game concludes when either the player has won, his opponent has won, or the game has ended in a draw. The time, correct to the nearest minute, for each game to conclude was noted and the results are given below.

Player A:	7	23	32	32	33	41	46	56	56	61	62
Player B:	3	7	8	8	14	26	27	37	37	41	58

(i) For player A's times, find

(a) the median, [1]

(b) the upper quartile, [1]

(c) the lower quartile. [1]

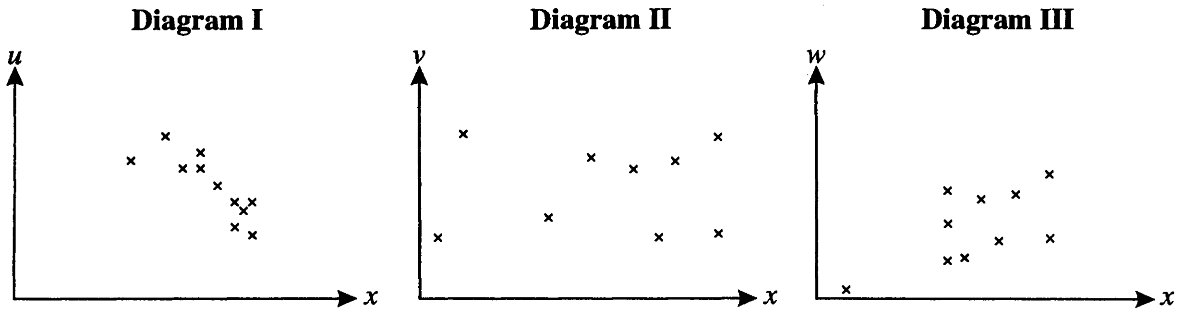
(ii) On graph paper, using the same axis, draw two box-and-whisker plots to represent the times of the games played by A and B. [6]

(iii) Use your diagram to make two statements comparing the two sets of data. [2]

(iv) Can you state from the diagram or from the data which player was the more successful? Give a reason for your answer. [1]

[Question 7 is printed overleaf.]

7 **(a)** The three scatter diagrams, I, II and III below, represent the way in which three variables, u, v and w, vary with respect to a fourth variable x.

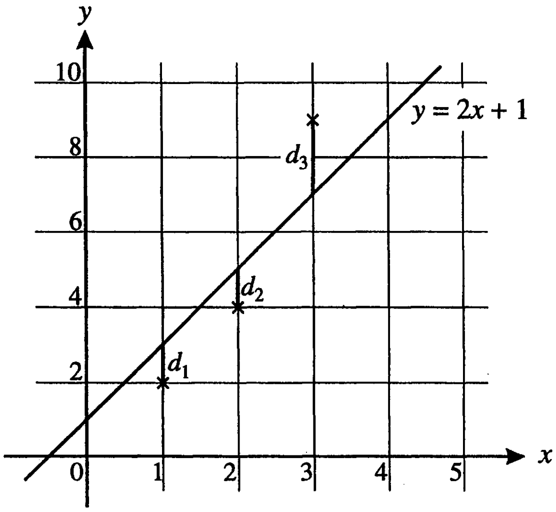


A student calculated the product moment correlation coefficient for each of the three sets of bivariate data and obtained the values

$$0.07, \qquad 0.62, \qquad -0.82.$$

State, with a reason in each case, which correlation coefficient corresponds to each of diagrams I, II and III. [4]

(b) To investigate the relationship between x and a new variable, y, the student collected three pairs of values of (x, y), namely $(1, 2)$, $(2, 4)$ and $(3, 9)$. The three corresponding data points are shown on the scatter diagram below, together with the student's attempt at a regression line which he fitted by eye. The equation of the student's line is $y = 2x + 1$.



(i) Calculate the sum of squares of the deviations of the three data points from the student's line. That is, calculate $S^2 = d_1^2 + d_2^2 + d_3^2$. [3]

(ii) Calculate the equation of the least squares regression line of y on x for the student's data and calculate the sum of squares of the deviations of the three data points from this line. [5]

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1

Candidate	A	B	C	D	E	F	G
Director 1	**5**	**6**	**4**	**3**	**1**	**7**	**2**
Director 2	6	5	2	4	3	7	1
d_i	-1	1	2	-1	-2	0	1

$$r_s = 1 - \frac{6\sum d_i^2}{n\left(n^2-1\right)} = 1 - \frac{6\times 12}{7\times 48} = \frac{11}{14} = \mathbf{0.786} \qquad \text{(3 s.f.)}$$ [5]

There is reasonably strong agreement between the assessments of the two directors. [1]

2 $\sum p_i = 1 \quad \Rightarrow \quad 0\cdot 2 + 0\cdot 1 + 0\cdot 1 + 0\cdot 4 + p = 1 \quad \Rightarrow \quad p = \mathbf{0\cdot 2}$ (show) [2]

$\mu = \sum x_i p_i = 2\times 0\cdot 2 \ + \ 3\times 0\cdot 1 \ + \ 4\times 0\cdot 1 \ + \ 5\times 0\cdot 4 \ + \ 6\times 0\cdot 2 = \mathbf{4\cdot 3}$ [2]

$\sigma^2 = \sum x_i^2 p_i \ - \ \mu^2 = \left(2^2\times 0\cdot 2 + ... + 6^2 \times 0\cdot 2\right) \ - \ 4\cdot 3^2 = \mathbf{2\cdot 01}$ [3]

3 no. of arrangements $= 6! = \mathbf{720}$ [1]

no. of arrangements with Mr. & Mrs. Aziz together $= 2\times 5! = \mathbf{240}$ [4]

$\text{p}\left(\text{Mr. \& Mrs. Aziz together}\right) = \frac{240}{720} = \mathbf{\frac{1}{3}}$ [2]

4 $\text{p}\left(\text{first disc is red}\right) = \mathbf{\frac{4}{15}}$ [1]

$\text{p}\left(2^{\text{nd}} \text{ disc is white} \mid 1^{\text{st}} \text{ disc is green}\right) = \frac{6}{14} = \mathbf{\frac{3}{7}}$ [1]

$\text{p}\left(\text{exactly two discs are red}\right) = 3\left(\frac{4}{15}\cdot\frac{3}{14}\cdot\frac{11}{13}\right) = \frac{66}{455} = \mathbf{0\cdot 145}$ (3 s.f.) [3]

$\text{p}\left(\text{all the same colour}\right) = \frac{4}{15}\cdot\frac{3}{14}\cdot\frac{2}{13} \ + \ \frac{6}{15}\cdot\frac{5}{14}\cdot\frac{4}{13} \ + \ \frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13} = \frac{34}{455} = \mathbf{0\cdot 0747}$ (3 s.f.) [3]

5 $\text{p}(< 3 \text{ throws}) = 1 - \text{p}(\geq 3) = 1 - \left(\frac{33}{35}\right)^2 = 0 \cdot 111020... = \mathbf{0 \cdot 111}$ (3 s.f.)

[3]

$\text{E}[\text{no. of throws}] = \frac{1}{p} = 1 \div \frac{2}{35} = \frac{35}{2} = \mathbf{17 \cdot 5}$

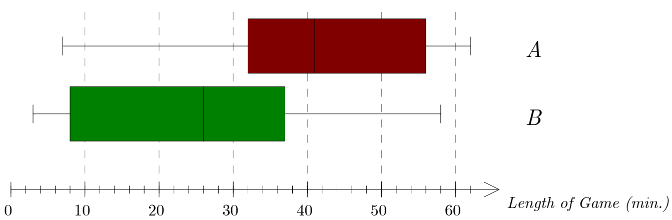
[2]

$\text{p}(\text{pays more than it's worth}) = \text{p}(> 40 \text{ throws}) = \left(\frac{33}{35}\right)^{40} = 0 \cdot 095024... = \mathbf{0 \cdot 0950}$ (3 s.f.)

[3]

6 player A

median = 6th value = **41'** U.Q. = 9th value = **56'** L.Q. = 3rd value = **32'** [1] [1] [1]



two statements of comparison [2]

time gives **no indication of success** of the players [1]

box plot [6]

7 $r_I = -0 \cdot 82$ $r_{II} = 0 \cdot 07$ $r_{III} = 0 \cdot 62$ (with a reason in each case) [4]

$$S^2 = \sum d_i^{\,2} = \left(2 - (2 \times 1 + 1)\right)^2 + \left(4 - (2 \times 2 + 1)\right)^2 + \left(9 - (2 \times 3 + 1)\right)^2 = 1 + 1 + 4 = \mathbf{6}$$ [3]

regression line of y on x

$$y - \overline{y} = \frac{\frac{\sum xy}{n} - \overline{x}\,\overline{y}}{\frac{\sum x^2}{n} - \overline{x}^2}(x - \overline{x}) \qquad y - 5 = \frac{2 \cdot 3333...}{0 \cdot 6666...}(y - 2) \qquad \boldsymbol{y = 3 \cdot 5\, y - 2}$$

$$S^2 = 0 \cdot 25 + 1 + 0 \cdot 25 = \mathbf{1 \cdot 5}$$

[5]

[Total 60]